

# Global topology of brain-wide co-fluctuations links task states, personality, and behavioral symptom dimensions

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## Abstract

Functional connectivity in network neuroscience is traditionally characterized using time-averaged correlations between brain regions. While these summaries capture stable large-scale organization, they do not fully reflect the temporal structure of moment-to-moment interactions. Here, we investigate how the order of interaction used to represent brain dynamics shapes the organization recovered from neural data. We compare three interaction representations of fMRI dynamics: regional activation (node time series), pairwise co-fluctuations (edge time series), and higher-order triplet interactions (triangle time series); within a common topological framework using Mapper from topological data analysis (TDA). Across task and resting-state data, Mapper representations derived from pairwise co-fluctuations more distinctly segregate task conditions than activation-based or higher-order representations. This organization reflects structured coordination patterns beyond activation polarity and is driven by high-amplitude interaction events. Beyond task states, modularity quality computed across all Mapper representations is highest for edge time series and selectively associated with stable individual differences: higher modularity relates to higher conscientiousness and lower internalizing and externalizing symptom dimensions. Together, these findings suggest that behaviorally relevant information is reflected in the topology of moment-to-moment brain interactions. Topological analysis of interaction-level dynamics therefore provides a complementary and interpretable framework for linking large-scale neural coordination to cognition, personality, and mental health.

## 1. Introduction

The brain can be conceptualized as a complex network of neurons that self-organizes into distinct brain regions characterized by relatively homogeneous cytoarchitecture and neural activity patterns. At the macroscopic scale, whole-brain function is often studied by examining coordinated activity among these regions over time. Functional connectivity (FC), typically quantified as the Pearson correlation between regional neural activity time series, is the most widely used approach for summarizing such coordination. While FC has proven valuable for characterizing large-scale functional organization of the brain (Bullmore & Sporns, 2009; Craddock et al., 2013; Rogers et al., 2007; Smith, 2012; Smith et al., 2009), it is inherently a time-averaged measure and therefore does not fully capture transient brain dynamics that may be behaviorally or clinically meaningful.

To overcome this limitation, recent work has introduced *edge time series*, which quantify moment-to-moment interactions between pairs of brain regions by computing the framewise product of their standardized activity signals, also referred to as regional co-fluctuations (Faskowitz et al., 2020; Owen et al., 2021; Zamani Esfahlani et al., 2020) (see Fig. 1d). Important to note that edge time series can be generalized beyond brain region pairs to triplets and groups of more brain regions (Santoro et al., 2024). Unlike traditional FC, edge time series and its higher-dimensional generalizations preserve temporal information and enable the investigation of rapid, transient coordination patterns across the brain. As such, they provide a more fine-grained representation of brain dynamics that may capture task-evoked and spontaneous state transitions. In particular, the edge time series is robust and carries subject-specific information suitable for neural fingerprinting (Sporns et al., 2021).

Analysis of edge time series has revealed the unique role of *high-amplitude frames*, which are brief and intermittent "events" of globally coordinated co-fluctuation (Faskowitz et al., 2020). These infrequent events disproportionately shape time-averaged functional connectivity. Their fundamental importance is underscored by findings that they are constrained by the anatomical connectome (Pope et al., 2021), are conserved across species (Ragone et al., 2024), and synchronize across individuals during shared movie watching (Tanner et al., 2023). High-amplitude frames therefore represent large-scale coordination events.

While edge time series provide a rich description of transient brain dynamics, their extreme high dimensionality poses a fundamental challenge for analysis and interpretation. Conventional dimensionality reduction techniques necessarily compress high-dimensional data into a low-dimensional space, often introducing information loss by collapsing distinct local structures. To address this limitation, we employ Mapper, a tool from topological data analysis that provides a principled approach for embedding high-dimensional data into a low-dimensional representation while preserving essential local relationships (Singh et al., 2007).

91 Mapper operates by covering the data in a low-dimensional space and performing  
92 partial clustering within overlapping regions of this space, resulting in a graph whose  
93 nodes represent locally coherent clusters of high-dimensional data points, and whose  
94 edges represent overlap of clusters (Singh et al., 2007). Importantly, this construction  
95 preserves local distances and neighborhood structure in the original high-dimensional  
96 space, while explicitly representing where information loss occurs during dimensionality  
97 reduction via node overlap and graph connectivity. In this way, Mapper represents  
98 trajectories through high-dimensional space as a network rather than a point-cloud  
99 projection, capturing global organization while preserving local geometric structure.

100  
101 Previous work has demonstrated that Mapper graphs can reveal meaningful structure in  
102 neural time series and that graph-derived features can be robustly related to individual  
103 differences in behavior and cognition (Geniesse et al., 2022, 2025; Rosenberg et al.,  
104 2023; Saggar, Bruno, et al., 2022; Saggar, Shine, et al., 2022; Saggar et al., 2018). In  
105 the context of edge time series, the resulting Mapper graph can be interpreted as a low-  
106 dimensional summary of the manifold along which brain-wide co-fluctuations evolve  
107 over time. To our knowledge, this is among the first applications of Mapper to edge time  
108 series data, offering a novel perspective on the temporal organization of brain-wide co-  
109 fluctuations.

110  
111 Here, we investigate whether edge-level dynamics provide a potentially more  
112 informative representation of brain state transitions than traditional activation-based  
113 (node-level) signals or even higher-order interaction signals (e.g., triangle-level). We  
114 compare node-, edge-, and triangle-level representations to determine how interaction  
115 order shapes recovered brain organization, and test whether global topological features  
116 derived from these representations relate to individual differences in personality and  
117 behavioral symptom dimensions. To this end, we construct Mapper graphs from  
118 subject-wise edge time series derived from parcellated, concatenated task and resting-  
119 state fMRI data from Human Connectome Project (HCP) participants (Barch et al.,  
120 2013; Smith et al., 2013).

121  
122 To quantify task-related organization within these Mapper graphs, we compute the  
123 quality of modularity with respect to task labels. Importantly, task labels are not used  
124 during Mapper graph construction; rather, they are assigned post hoc to nodes based  
125 on the dominant proportion of time frames they contain and are used only to evaluate  
126 the degree of segregation across tasks. The quality of modularity is then computed  
127 using this predefined partition, without performing community detection on the graph  
128 itself. This measure therefore quantifies the extent to which nodes segregate according  
129 to task identity (Newman, 2006; Rubinov & Sporns, 2010). We compare the quality of  
130 modularity across Mapper graphs constructed from edge time series, traditional node-  
131 level activation time series, and triangle-level time series.

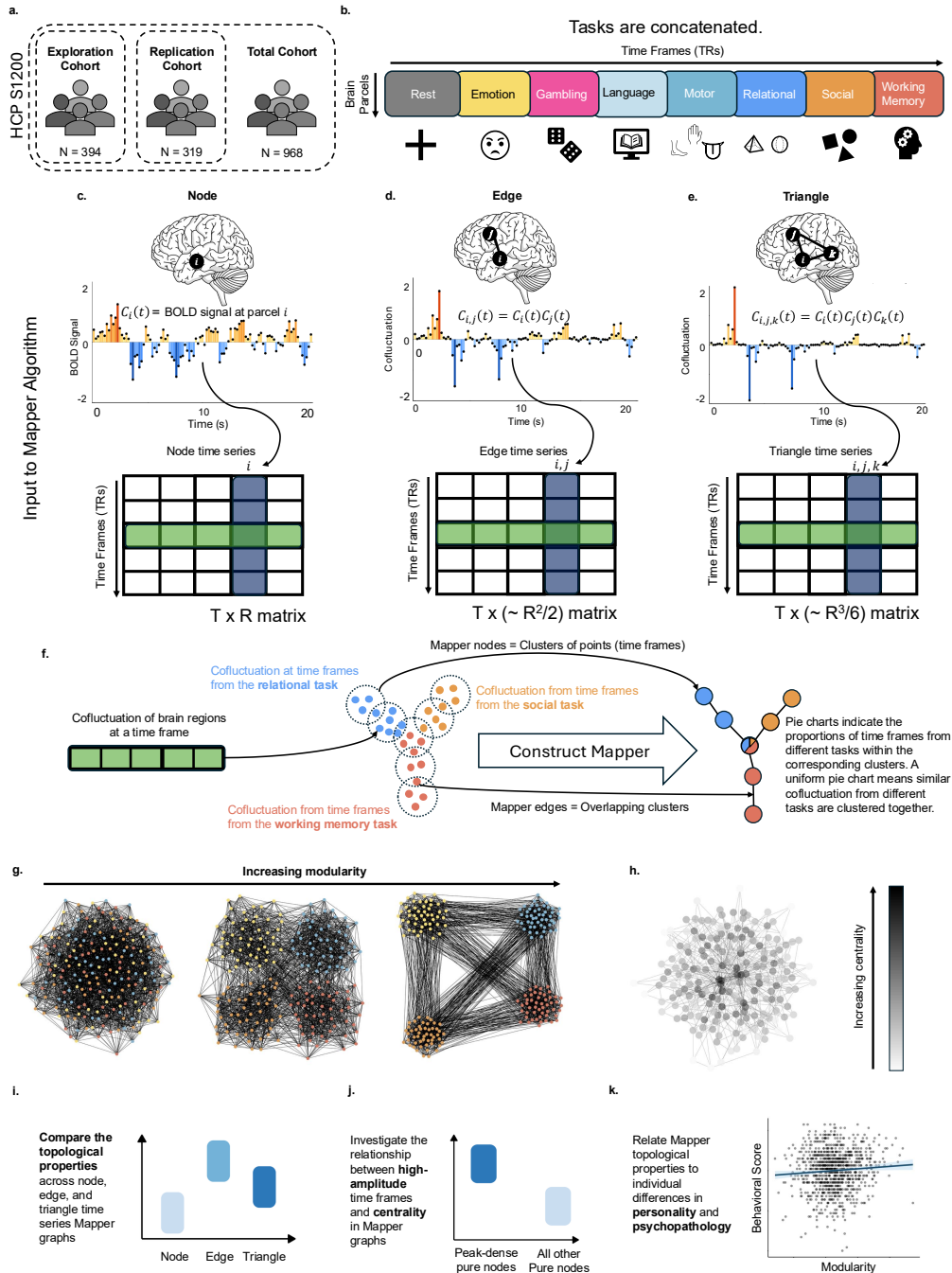
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133 Using this framework, we find that Mapper graphs derived from edge time series exhibit  
134 structured organization, and they show greater task segregation than graphs derived  
135 from node- or triangle-based representations. Moreover, global topological properties of  
136 these graphs are associated with individual differences, with the quality of modularity for

137 the edge time series relating to personality traits and self-reported symptom  
138 dimensions.

139  
140 To investigate the origin of the global organization in edge time series Mapper graphs,  
141 we analyze the contribution of high-amplitude frames and the mathematical effect of the  
142 construction of the edge time series on the distances between activation/co-fluctuation  
143 vectors. We identify a prominent role for peak-dense pure nodes, i.e., task-homogenous  
144 nodes that are dense with high-amplitude frames, in increasing the quality of modularity,  
145 as shuffling their task labels leads to a significant drop in the quality of modularity. We  
146 further observe that functional connectivity driven by these nodes corresponds to  
147 activation of neurobiologically meaningful, task-specific large-scale brain systems. Our  
148 mathematical analysis reveals the quadratic relationship between the node-level  
149 distance and edge-level distance between brain (co-) activation vectors. This suggests  
150 that the construction of edge time series emphasizes relational structure over absolute  
151 activation, making task-specific coordination patterns more coherent in the Mapper  
152 graph.

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154 Taken together, these results introduce a framework for visualizing and analyzing  
155 dynamic brain interactions and suggest that the global topology of brain-wide co-  
156 fluctuations relates to stable behavioral variation.

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**Fig. 1: Overview of the approach.** [a] The Human Connectome Project Young Adult (HCP-YA) dataset was analyzed. After excluding subjects with incomplete data, 968 participants were retained and split into an exploration cohort (N = 394) and an independent replication cohort (N = 319). Behavioral associations were evaluated in the full combined cohort (N = 968). [b] For each participant, parcellated BOLD time series were concatenated across resting-state and task fMRI scans. [c] Each column of the resulting matrix corresponds to a z-scored regional activation (node) time series for a single brain parcel. [d] Edge time series were constructed as the framewise product of pairs of node time series, capturing moment-to-moment co-fluctuations between brain regions. [e] Triangle time series were similarly constructed as the framewise product of triplets of node time series, representing higher-order interactions. [f] Construction of a Mapper graph from the edge time series for illustration. A similar pipeline was used for node- and triangle time series. Each row of the edge time series matrix (a co-fluctuation vector at a single time frame) is treated as a data point in high-dimensional space. Mapper summarizes the geometry of these data via local clustering, producing a graph whose nodes represent clusters of time frames and whose edges indicate overlapping

172 clusters. Pie charts indicate the proportion of time frames from different tasks within each Mapper node. [g]  
173 Illustration of graph modularity. Graphs in which nodes of the same type (indicated by color) preferentially cluster  
174 together exhibit a higher quality of modularity. [h] Illustration of node centrality, with darker shading indicating higher  
175 centrality (greater proximity to the average node in the graph). [i–k] Schematic of the analysis pipeline, comparing  
176 topological properties across Mapper graphs constructed from node, edge, and triangle time series [i], examining the  
177 relationship between high-amplitude (peak) time frames and node centrality [j], and relating Mapper-derived  
178 topological properties to individual differences in personality and behavioral symptom scales [k].

179

## 180 2. Results

181

182 We sought to characterize the structure of moment-to-moment brain coordination using  
183 representations defined at multiple interaction orders. For each participant, resting-state  
184 and task fMRI data were parcellated (Schaefer et al., 2018), z-scored, and  
185 concatenated, and each time frame was represented as (i) regional activation patterns  
186 (node time series), (ii) pairwise co-fluctuations between regions (edge time series), and  
187 (iii) triplet interactions (triangle time series) (Fig. 1c–e). For each representation, we  
188 constructed a Mapper graph that groups similar time frames into locally coherent  
189 clusters, producing a low-dimensional network representation of brain activity  
190 trajectories (Fig. 1f).

191

192 Using these parallel representations within a common Mapper framework, we  
193 addressed three questions: (a) how interaction order affects task structure, (b) which  
194 signal features generate the observed organization, and (c) whether global topological  
195 properties relate to stable individual differences. The following sections compare task  
196 segregation across representations, examine the role of high-amplitude frames and  
197 their anatomical anchors, characterize relationships between node- and edge-level  
198 geometry, test associations with personality and behavioral symptom dimensions, and  
199 assess reliability across sessions.

200

### 201 2.1 The edge time series is better at task profiling than node and triangle time 202 series.

203

204 We first examined whether different interaction orders provide distinct representations of  
205 task structure. Mapper graphs constructed from edge time series exhibited a consistent  
206 ring-like organization, in which peripheral nodes were dominated by specific task  
207 conditions while centrally located nodes contained mixtures of task time frames (Fig.  
208 2b). For example, resting-state frames preferentially occupied one region of the ring,  
209 whereas working memory and social task frames localized to another. This organization  
210 was consistently observed across participants (Fig. S3).

211

212 In contrast, Mapper graphs derived from node time series showed weak separation  
213 between tasks (Fig. 2a). Triangle time series preserved aspects of the ring-like  
214 geometry but displayed reduced task specificity compared to edge time series (Fig. 2c).

215

216 To quantify task segregation, we computed modularity quality using task identity as a  
217 predefined partition of nodes. Each node was assigned a task label based on the  
218 dominant proportion of time frames it contained. Across participants in the exploration  
219 cohort (N = 394), edge time series showed higher modularity quality than both node and

220 triangle representations (Fig. 2d; paired t-tests,  $t = 59.0, 17.6, df = 393$ , Bonferroni-  
221 corrected  $p < 0.001$ , see Table S1 for complete statistics). This result replicated in an  
222 independent cohort (Fig. 2e).

223  
224 To rule out the effect of scan session and parcellation, we also reproduced this finding  
225 in the exploration cohort with a different scan session (RL as opposed to LR), and with a  
226 different parcellation (Schaefer 200 instead of Schaefer 100 parcels) (Fig. S1). These  
227 results indicate that, as measured by modularity quality, the edge time series shows  
228 stronger task segregation than the node and triangle representations across acquisition  
229 and parcellation choices.

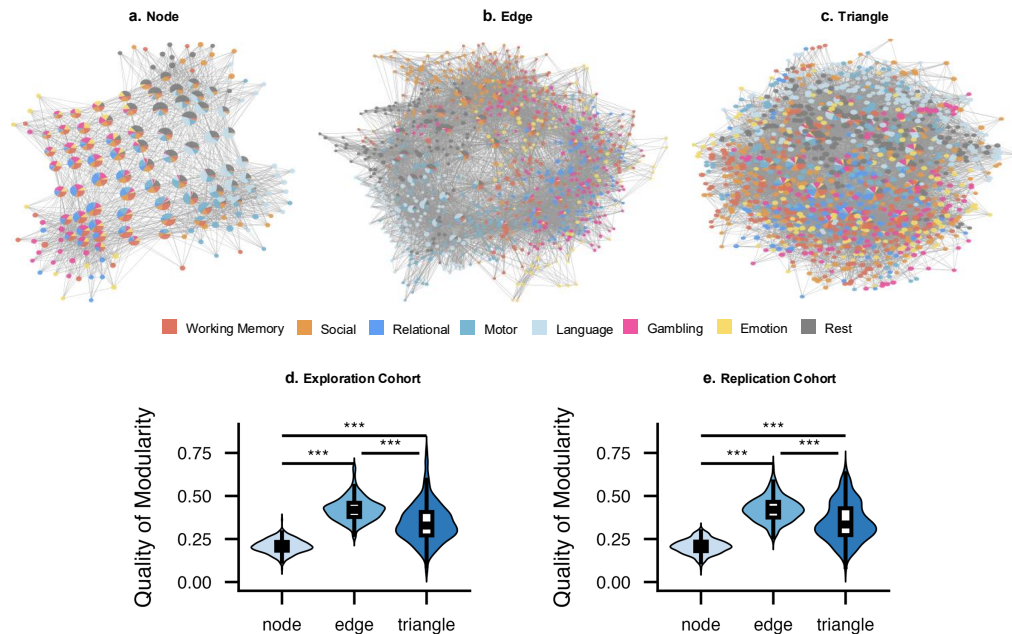
230  
231 We also carried out a separate analysis to rule out the effect of simpler graph statistics  
232 like the number of Mapper nodes and Mapper edges. When these statistics are  
233 controlled, edge time series still attains the highest mean quality of modularity (see  
234 Supplementary Materials Section S2).

235  
236 Because edge time series have substantially higher dimensionality than node time  
237 series, we tested whether dimensionality alone accounted for this difference. We  
238 projected all representations onto top principal components prior to Mapper  
239 construction, yielding comparable feature dimensionality across representations (Fig.  
240 S1e – h, Table S1). This procedure reduced the modularity differences, but the  
241 differences are still significant ( $ps < 0.001$ ), indicating that dimensionality contributes to,  
242 but does not fully explain, the stronger task segregation observed for edge time series.

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244 These results establish that interaction order influences the degree of task segregation  
245 recovered by the Mapper representation.

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247 We next examined which properties of the time series give rise to this organization.  
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 252 **Fig. 2: The edge time series is the best at task profiling.** [a – c] Mapper graphs for the node, edge, and triangle  
 253 time series of a representative individual. [d – e] Distributions of the quality of modularity of edge time series Mapper  
 254 graphs of the exploration cohort and the replication cohort. See Table S1 for complete statistics. \*\*\* means  
 255 Bonferroni-corrected p-values < 0.001.

256  
 257 **2.2 High-amplitude (peak) frames drive task segregation by potentially acting as**  
 258 **hubs**

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 260 Previous works using edge-time series revealed that the brain-wide co-fluctuations  
 261 occur in intermittent high-amplitude bursts (Faskowitz et al., 2020). These time points,  
 262 termed high-amplitude or peak frames, correspond to moments in which many edges  
 263 simultaneously exhibit large co-fluctuations and disproportionately contribute to time-  
 264 averaged functional connectivity (Zamani Esfahlani et al., 2020). Under naturalistic  
 265 stimulation, such events align across individuals (Tanner et al., 2023).

266  
 267 We therefore examined whether peak frames contributed to the task segregation  
 268 observed in Section 2.1. Specifically, we focused on pure Mapper nodes, i.e., nodes in  
 269 which >75% of time frames belong to a single task, because mixed nodes might act as  
 270 bridges between tasks and could reduce segregation. The choice of 75% as the purity  
 271 threshold did not affect the results, as less than 1% of all nodes have purity level strictly  
 272 between explained in 60% and 100% (see Supplementary Materials Section S6 for  
 273 details). For the representative participant shown in Fig. 2b, the distribution of peak  
 274 frames across the Mapper graph is shown in Fig. 3c, and pure nodes with the highest  
 275 proportion of peak frames (referred to later as peak-dense pure nodes) are highlighted  
 276 in Fig. 3d.

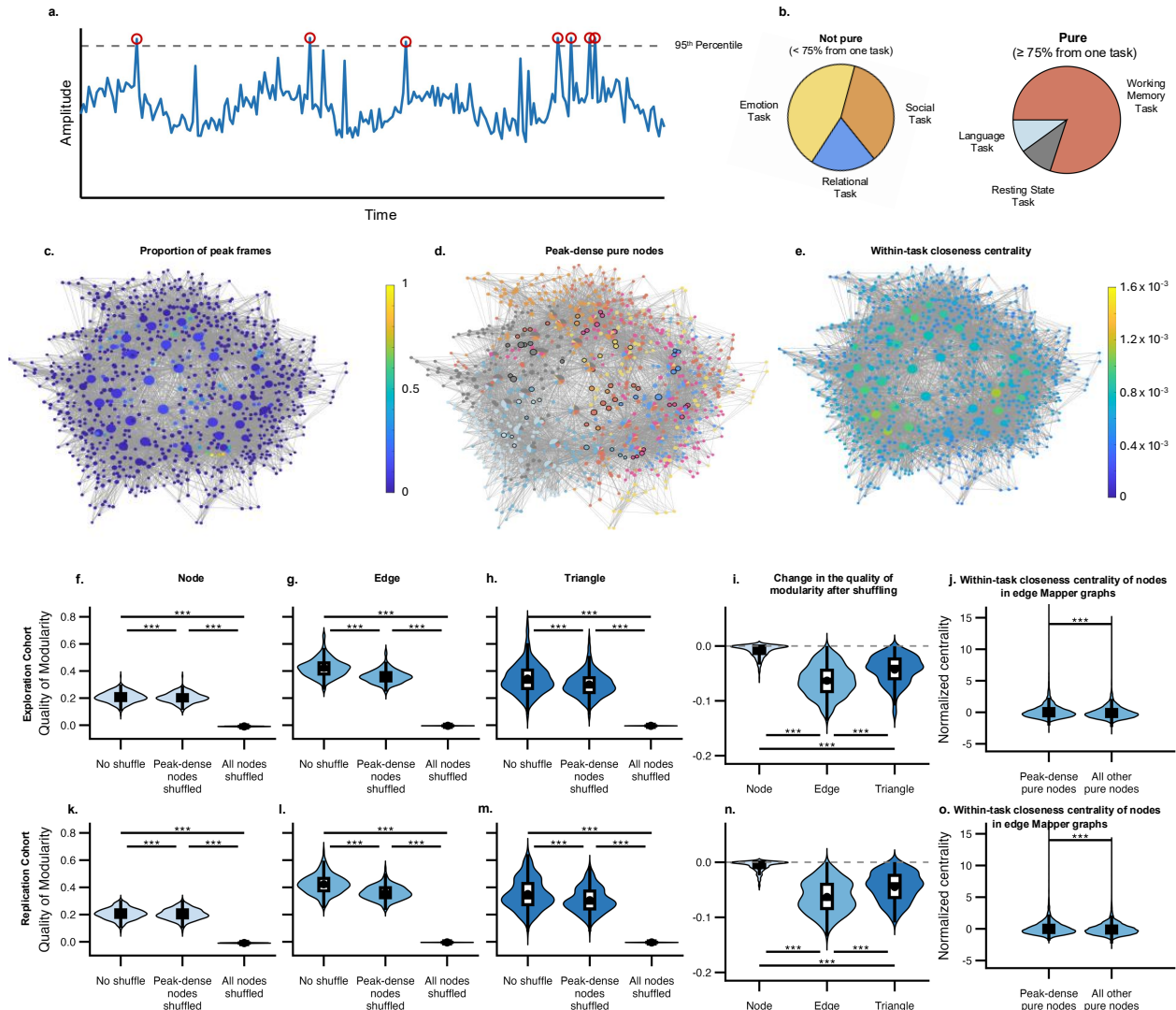
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278 To test peak-dense pure nodes' contribution to graph organization, we selectively  
279 shuffled task labels while preserving graph topology. Four conditions were compared:  
280 no shuffling, shuffling peak-dense pure nodes only, shuffling all nodes, and shuffling a  
281 matched number of randomly selected nodes. Complete statistics are presented in  
282 Table S3. Shuffling only peak-dense pure nodes produced the largest reduction in  
283 modularity quality in edge time series graphs, significantly greater than in node or  
284 triangle representations. This reduction exceeded that observed when shuffling an  
285 equal number of randomly selected nodes, indicating a specific contribution of peak-  
286 dense nodes. Similar effects were observed at the individual-subject level (Fig. 3i; Fig.  
287 S5). Shuffling all nodes eliminated modularity quality across representations (Fig. 3f–h),  
288 showing that the measured modularity reflects alignment between graph structure and  
289 task identity.

290  
291 To understand why these peak-dense nodes exert such influence, we examined their  
292 within-task centrality. Peak-dense pure nodes frequently occupied central positions  
293 within task-specific clusters (Fig. 3d), suggesting a hub-like role. Thus, we hypothesized  
294 that they drive up the quality of modularity by being hubs of their respective tasks, and  
295 we tested this hypothesis by measuring the within-task centrality of peak-dense pure  
296 nodes. Quantitative analysis confirmed that peak-dense nodes had significantly higher  
297 centrality than other pure nodes ( $t = 20.8$ ,  $p = 1.43E-94$ ; Fig. 3j, Table S4), and this  
298 effect remained significant in a mixed-effects model accounting for subject variability ( $t =$   
299  $93.9$ ,  $p = 0$  (too small to be estimated), Table S5).

300  
301 All findings replicated in an independent cohort (Fig. 3k–o). Controlling for head motion  
302 gives similar results (Tables S5 and S7).

303  
304 Together, these results show that a small subset of high-amplitude frames  
305 disproportionately contributes to task segregation in edge time series Mapper graphs.



307  
 308 **Fig. 3. Peak frames drive task segregation as pure peak-dense Mapper nodes serve as task hubs.** [a] Peak  
 309 frames are time frames above the 95th percentile in amplitude. [b] In a pure Mapper node, over 75% of the  
 310 time frames in the node belong to the same task. A pure Mapper node is said to be peak-dense if its proportion of  
 311 peak frames is above the 90th percentile. [c] The edge time series Mapper graph in Fig. 2b, colored by the proportion of  
 312 peak frames. [d] The same Mapper graph with peak-dense pure Mapper nodes highlighted. [e] The same Mapper  
 313 graph colored by the within-task centrality. [f – h] Distributions (violin plots) of the quality of Mapper modularity with  
 314 task labels shuffled (not shuffled, only labels of peak-dense pure Mapper nodes shuffled, and all labels shuffled)  
 315 across simplices (node, edge, triangle) for the exploration cohort. [i] Distributions of the change in the quality of  
 316 modularity upon shuffling task labels of peak-dense pure Mapper nodes across simplices. [j] Distributions of within-  
 317 task centrality of peak-dense pure Mapper nodes and other pure Mapper nodes for the exploration cohort. [k – o]  
 318 Counterparts of Fig. [f – j] for the replication cohort. Throughout all violin plots, pairwise t-tests are done with  
 319 Bonferroni correction, and asterisks indicate significance (\* corrected  $p < 0.05$ , \*\* corrected  $p < 0.01$ , \*\*\* corrected  $p < 0.001$ ).  
 320 See Tables S3 – S5 for complete statistics.

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322

### 2.3 Task-specific peak frames map onto task-relevant brain systems

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324

Having established that peak-dense pure nodes drive task segregation, we next asked whether these frames correspond to meaningful neurobiological organization.

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326

327 We first compared functional connectivity (FC) patterns derived from three subsets of  
328 time frames: all frames (traditional FC), frames belonging to peak-dense pure nodes,  
329 and peak frames alone.

330

331 Across tasks, the cohort-wide mean connectivity estimates from all three subsets were  
332 highly correlated (cohort-wide mean  $r > 0.719$ , lower bound of 95%-confidence interval  
333  $> 0.712$  across connectivity measures and tasks; see Fig. 4a and Table S9).

334 Connectivity derived from peak-dense pure nodes was generally more similar to  
335 traditional FC than connectivity derived from peak frames alone. This suggests that the  
336 dominant structure of functional connectivity is carried not simply by high-amplitude  
337 events, but by high-amplitude events that occur in consistent task-specific  
338 configurations.

339

340 Consistent with this, the spatial structure of the connectivity matrices was preserved  
341 across these estimates (Fig. 4b). Traditional FC exhibited the lowest edge magnitudes,  
342 peak-frame FC the highest, and peak-dense-node FC formed an intermediate pattern,  
343 suggesting that peak-dense nodes capture the dominant large-scale organization of  
344 connectivity while excluding low-amplitude fluctuations.

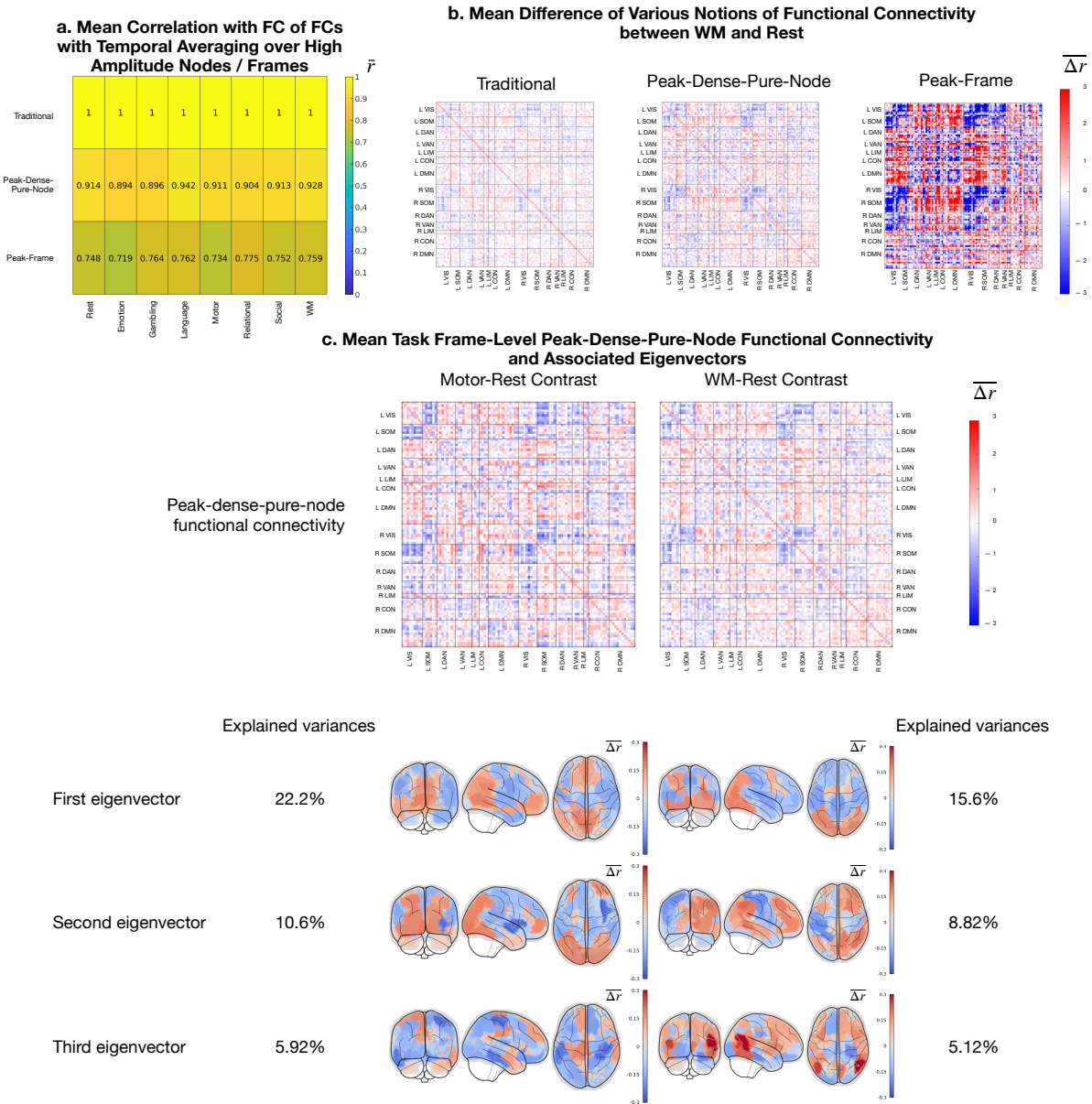
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346 We next tested whether these connectivity patterns correspond to task-relevant brain  
347 systems. Principal components of peak-dense-pure-node connectivity matrices were  
348 projected onto the cortical surface (Fig. 4c). During the motor task, a component  
349 emphasized the somatomotor network with opposing hemispheric signs consistent with  
350 asynchronous limb movements. During the working-memory task, a corresponding  
351 component highlighted frontoparietal regions, whereas earlier components reflected  
352 general task engagement (e.g., visual involvement). Similar results were observed in  
353 the replication cohort (Fig. S9), and approximate equivalence of eigenvectors across  
354 scaled matrices is shown in Fig. S8.

355

356 Together, these findings indicate that peak-dense frames that organize Mapper  
357 topology correspond to structured, task-relevant large-scale brain systems rather than  
358 arbitrary fluctuations.

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360  
 361 **Fig. 4: Anchoring task-specific peak frames into task-relevant brain regions.** [a] Each row is a notion of  
 362 functional connectivity defined by taking temporal average over a choice of high amplitude nodes or frames, and each  
 363 column is a task. The colors of the heatmap show the correlation of different forms of functional connectivity with the  
 364 traditional functional connectivity, averaged across subjects in the exploration cohort. The overall yellowish tone of  
 365 the heatmap shows the correlations are all close to 1, indicating a linear relationship between different forms of  
 366 functional connectivity with the traditional functional connectivity. [b] The mean difference of functional connectivity  
 367 matrices between the working memory task and the resting state in the discovery cohort. The more selective we are  
 368 for high amplitude, the higher the intensity of the heatmap. [c] The mean difference in peak-frame functional  
 369 connectivity matrices from rest for the motor and the working memory tasks and its top 3 eigenvectors (principal  
 370 components). Note that in the third eigenvector, there are strong (very positive or very negative) loadings in the  
 371 somatomotor network for the motor task, and strong loadings in the frontoparietal control network for the working  
 372 memory task.

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## 2.4 Quadratic Relationship between Node-Level and Edge-Level Distances

To understand why edge time series produce a different graph organization than node time series, we examined how similarity between time frames is defined in each representation. In Mapper, clustering depends on distances between brain states: node time series compare activation vectors, whereas edge time series compare co-fluctuation vectors (Fig. 1c-d).

We begin with a qualitative examination of pairwise distances between time frames across representations. For a representative participant, these distances differed markedly (Fig. 5a). Node-based distances exhibited a checkerboard structure, indicating that some time frames were extremely dissimilar. In contrast, edge-based distances were compressed and lacked these extremes. This suggests that time frames considered opposites in activation space are treated as relatively similar in co-fluctuation space.

Plotting the two distances against each other revealed a striking nonlinear relationship (Fig. 5b). Across time-frame pairs, the edge-based distance followed a quadratic mapping of the node-based distance:

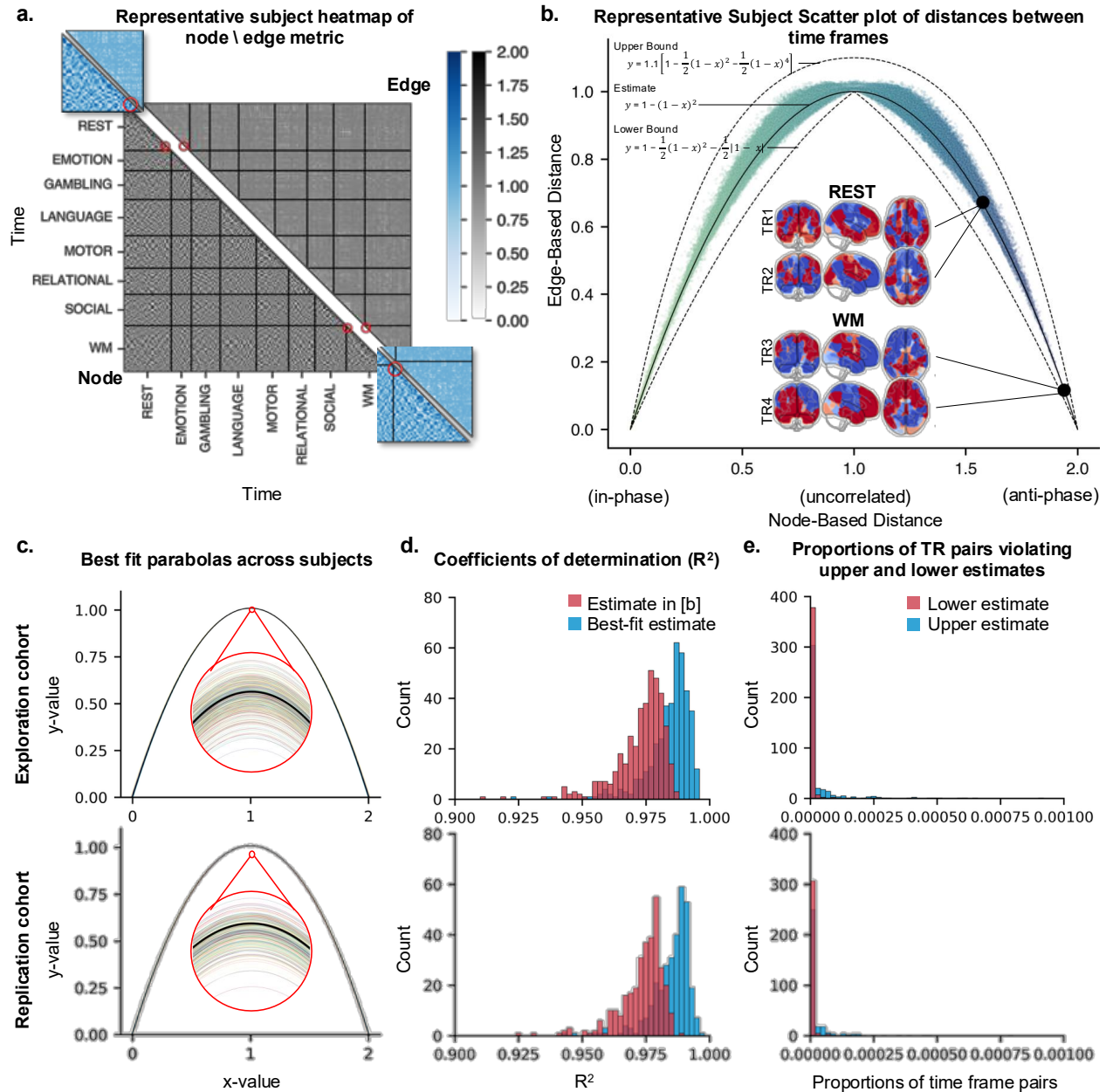
$$dist_{edge}(time_1, time_2) \approx 1 - (1 - dist_{node}(time_1, time_2))^2$$

Because correlation distance ranges from 0 (identical) to 2 (anti-phase), this mapping preserves similarity at small distances but compresses large distances, causing anti-correlated activation patterns to appear close in edge space. Thus, edge representations emphasize similarity of interaction structure rather than similarity of activation amplitude.

The approximation in the above quadratic equation is validated in two ways. First, an ordinary least square quadratic approximation is taken for each subject in the exploration cohort, and the best-fit parabolas are all very close to the model parabola  $y = 1 - (1 - x)^2$  (Fig. 5c), and the coefficients of determination  $R^2$  for the best-fit parabolas and the model parabola are both very close to 1 (cohort-wide minimum  $R^2 = 91.0\%$ , Fig. 5d). The quadratic approximation can further refined by the upper bound  $y = 1.1 \left[ 1 - \frac{1}{2}(1 - x)^2 - \frac{1}{2}(1 - x)^4 \right]$  and the lower bound  $y = 1 - \frac{1}{2}(1 - x)^2 - \frac{1}{2}|1 - x|$ , as exemplified by one subject in Fig. 5b. The degree-4 term and the absolute value term are chosen to capture the distribution of points near  $x = 1$ , as the lower envelope of the points in Fig. 5b has a sharp kink, and the flatness of the top of the upper envelope suggests the need of a higher power term. The proportion of time point pairs violating either bound is again very low across subjects (cohort-wide maximum proportion of violation = 0.277%, Fig. 5e). These results are replicated in the replication cohort (315 subjects, cohort-wide minimum  $R^2 = 92.5\%$ , cohort-wide maximum proportion of violation = 0.277%).

422 For illustration, the most anti-phase activation pairs occurred during the working  
423 memory task and during rest (Fig. 5b), yet these pairs remained close in edge space,  
424 consistent with the distance transformation.

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426 Together, these findings show that the edge transformation reshapes the geometry of  
427 brain states by collapsing oppositely signed activation patterns into nearby points,  
428 thereby altering the topology recovered by Mapper. This helps explain the enhanced  
429 task segregation observed in Section 2.1: node-based representations treat opposite  
430 activation patterns as maximally dissimilar even when they share the same coordination  
431 structure, whereas edge representations preserve interaction relationships while  
432 discarding polarity. By reducing variability due to global sign and amplitude fluctuations,  
433 the representation emphasizes relational structure over absolute activation, making  
434 task-specific coordination patterns more coherent in the Mapper graph.  
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**Fig. 5: The quadratic relationship between the edge-based distance and node-based distance means that antiphase brain maps are considered similar under the edge-based distance. [a]** Heatmap of node-based distances (bottom-left) and edge-based distances (top-right) for brain maps at different time frames for a representative subject. The red circles are centered at time frame pairs illustrated in panel [b]. **[b]** Scatter plot of the node-based distances and edge-based distances of time frame pairs for the same subject, with illustrations of the normalized brain maps at time frame pairs furthest apart under the node-based distance (in other words, maximally anti-phase) across tasks and within the resting session. **[c]** The best-fit parabolas for the scatter plots in [b] for all subjects, along with the model parabola  $y = 1 - (1 - x)^2$ . **[d]** The coefficients of determination of the best-fit parabolas in [c] (blue) and of the model parabola (red). **[e]** The proportion of time frame pairs that violate the upper bound (blue) and the lower bound (red) in [b].

## 2.5 Edge Mapper modularity relates to personality and behavioral symptom scales

We next asked whether the global topological organization of edge time series relates to stable individual differences in behavior. For each participant, the quality of modularity was averaged across runs to reduce measurement variability. We then tested associations between edge modularity (the quality of modularity of an edge time series Mapper graph) and eight behavioral measures, including the five NEO-FFI personality domains, internalizing and externalizing symptom dimensions from the Adult Self-Report (ASR), and fluid intelligence (PMAT).

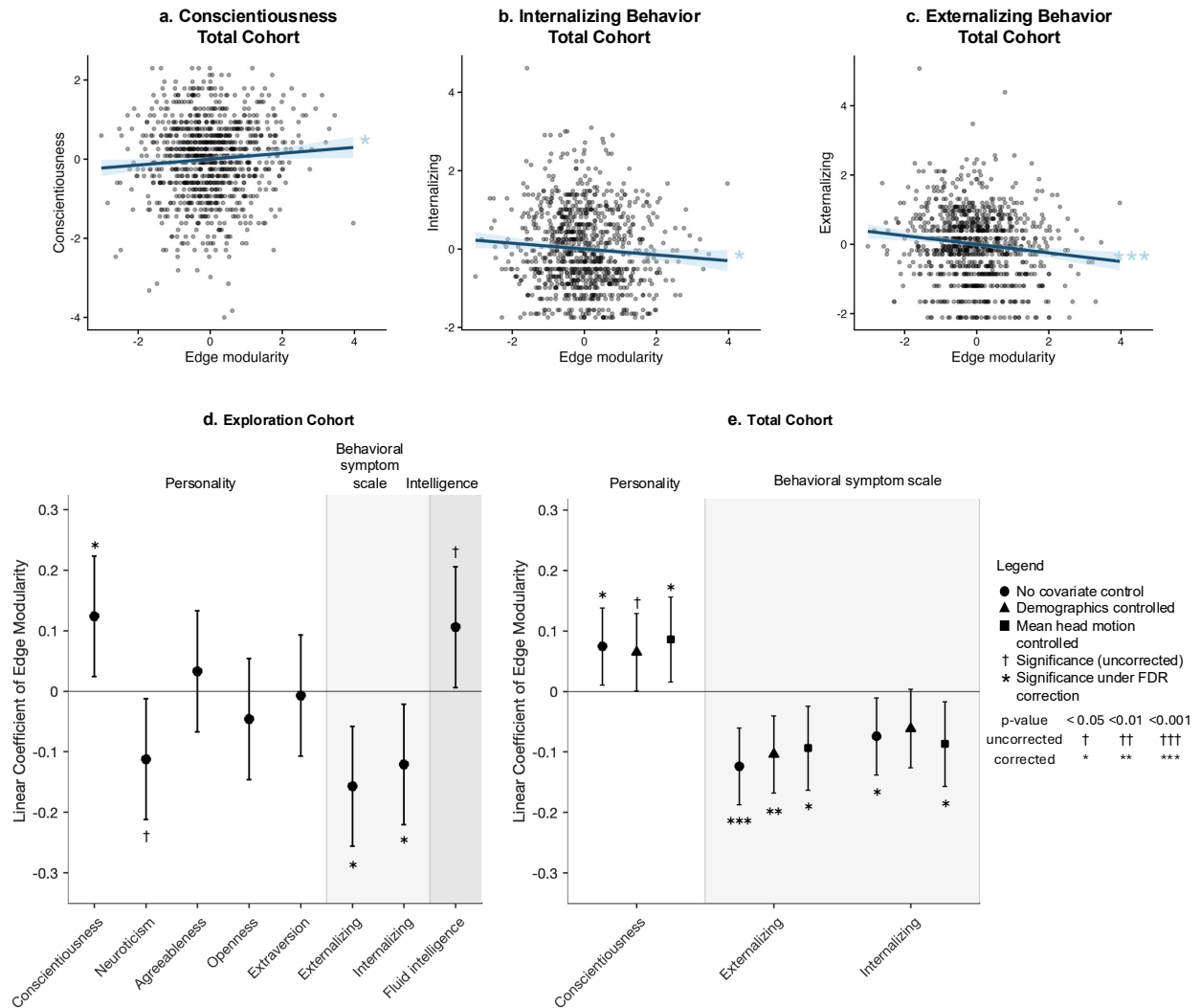
In the exploration cohort, the quality of modularity showed significant associations with conscientiousness ( $\beta = 0.124$ , uncorrected  $p = 0.0147$ ), neuroticism ( $\beta = 0.112$ , uncorrected  $p = 0.0274$ ), internalizing behavior ( $\beta = 0.121$ , uncorrected  $p = 0.0173$ ), and externalizing behavior ( $\beta = -0.157$ , uncorrected  $p = 0.00194$ ), and fluid intelligence ( $\beta = 0.106$ , uncorrected  $p = 0.0375$ ). Among them, the associations with conscientiousness (corrected  $p = 0.0461$ ), internalizing behavior (corrected  $p = 0.0461$ ) and externalizing behavior (corrected  $p = 0.0155$ ) are significant after Benjamini-Hochberg FDR correction. See Table S10 for complete statistics. These three relationships remained significant in after controlling for demographic variables (age and sex), and associations with conscientiousness persisted after accounting for head motion, while the internalizing and externalizing associations were reduced but remained comparable in magnitude (corrected  $p = 0.0708$  and corrected  $p = 0.0759$  respectively, Fig. S11b). See Table S11 for complete statistics for the analysis of the exploration cohort with control variables.

To improve statistical power and reduce behavioral noise, we repeated the analysis in the total cohort using mixed-effects models accounting for family structure, demographics, and head motion. In this analysis, conscientiousness, and internalizing and externalizing symptoms remained significantly associated with the quality of modularity after FDR correction, in the absence of covariate control and when head motion is controlled. When demographics (including family structure) is controlled, the association with externalizing behavior remains significant under FDR correction, those with conscientiousness ( $p = 0.0635$ ) and internalizing behavior ( $p = 0.0635$ ) approaches significance under FDR correction. Scatter plots and confidence intervals of regression coefficients are shown in Fig. 6. Complete statistics for the full cohort are presented in Table S12.

For comparison, we performed the same analysis using node and triangle time series Mapper graphs (Figs. S10 – S12, Tables S10 – 12). The quality of modularity of node time series Mapper graphs was significantly correlated with agreeableness (without covariate control:  $\beta = 0.0734$ , corrected  $p = 0.0246$ , see Fig S12a and Table S12) and fluid intelligence (without covariate control:  $\beta = 0.108$ , corrected  $p = 0.00179$ , see Fig S12a and Table S12), but neither correlation remained significant upon controlling for head motion. For triangle time series Mapper graphs, without covariate control, as well as when demographics (including family structure) is controlled, the association of the

497 quality of modularity and externalizing behavior (beta = -0.098, corrected p = 0.0054 (no  
 498 covariate control) and 0.00264 (demographics controlled)), was significant, but it loses  
 499 significance upon controlling for head motion (corrected p = 0.115). See Fig S12c and  
 500 Table S12 for complete statistics.

501  
 502 These findings indicate that global topological organization of moment-to-moment co-  
 503 fluctuations relates to individual differences in personality and mental health.



504 **Fig. 6: Behavioral correlates of edge modularity** (i.e. the quality of modularity for edge time series Mapper  
 505 graphs). **[a-c]** Edge modularity is significantly correlated with conscientiousness, a domain of temperament in the  
 506 NEOFFI model, as well as self-reported behavioral symptom scales and fluid intelligence in the total cohort. **[d]** 95%  
 507 confidence intervals of the linear coefficients of edge modularity in the exploration cohort. **[e]** 95% confidence  
 508 intervals of the linear coefficients of edge modularity in the total cohort with demographics (age, sex, and family) or  
 509 head motion controlled.  
 510  
 511  
 512  
 513  
 514  
 515

### 516 3. Discussion

517

518 Here we asked how the order of interaction used to represent brain activity shapes the  
519 latent organization recovered from neural dynamics. By comparing node-, edge-, and  
520 triangle-based representations within a common topological framework, we found that  
521 pairwise co-fluctuations (edge time series) yield more coherent task-dependent  
522 structure than activation-based or higher-order representations. This organization is  
523 closely associated with high-amplitude frames and can be understood geometrically  
524 through a nonlinear transformation that emphasizes interaction structure over activation  
525 polarity. The resulting topological property, i.e., modularity quality, is associated with  
526 individual differences in personality and symptom scales and shows stability across  
527 sessions. Together, these findings suggest that the topology of dynamic brain  
528 interactions provides an interpretable representation linking large-scale coordination  
529 patterns to behavior.

530

#### 531 3.1 Interaction order in large-scale brain dynamics

532

533 Complex affective and cognitive phenomena emerge from coordinated activity across  
534 distributed brain regions. Traditional node-level analyses describe activity within  
535 regions, but they may not capture how relationships between regions structure brain  
536 dynamics. This has motivated increasing interest in interaction-based representations,  
537 although the appropriate interaction order remains debated. Some studies  
538 operationalize interactions using pairwise co-fluctuations such as edge time series and  
539 edge-based connectivity (Faskowitz et al., 2020; Zamani Esfahlani et al., 2020),  
540 whereas others emphasize triplets or larger collections of regions (Guo et al., 2021;  
541 Huang et al., 2017; Owen et al., 2021; Santoro et al., 2024). In parallel, different  
542 mathematical frameworks have been proposed to characterize these relationships,  
543 including topological approaches such as persistent homology (Petri et al., 2014; Yoon  
544 et al., 2024).

545

546 By directly comparing node-, edge-, and triangle-level representations within the same  
547 analytical framework, we observed that pairwise co-fluctuation structure yielded the  
548 most coherent task organization. Rather than indicating that higher-order interactions  
549 are uninformative, this finding suggests that interaction order introduces a trade-off  
550 between representational expressivity and noise amplification. Pairwise interactions  
551 appear to capture dominant coordination patterns while remaining sufficiently  
552 constrained to preserve stable structure, whereas higher-order interactions may require  
553 stronger regularization or alternative summaries to reveal consistent organization.

554

555 In this view, our results reconcile differing perspectives in the literature: large-scale  
556 brain dynamics contain higher-order structure, but pairwise interaction representations  
557 provide a practical level at which this latent organization becomes detectable in typical  
558 neuroimaging data.

559

#### 560 3.2 High-amplitude frames as task-specific hubs in dynamic brain topology

561

562 A growing body of work suggests that functional connectivity is disproportionately  
563 shaped by brief high-amplitude events in the edge time series (Ragone et al., 2024;  
564 Tanner et al., 2023; Zamani Esfahlani et al., 2020), whereas lower-amplitude frames  
565 more closely reflect canonical network structure (Betzel, Cutts, et al., 2023). Previous  
566 works revealed that the origins of these high-amplitude frames can be traced to the  
567 modular structure of the underlying structural connectome (Pope et al., 2021), and their  
568 amplitude is directly related to time-resolved structure-function coupling (Liu et al.,  
569 2022). These network dynamics are also linked to endogenous physiological processes,  
570 such as hormone fluctuations over the menstrual cycle (Greenwell et al., 2023).  
571 Importantly, these co-fluctuation events can be partitioned into discrete states that are  
572 stable within individuals (Betzel et al., 2022) and possess a hierarchical organization  
573 (Betzel, Cutts, et al., 2023), highlighting their role in shaping person-specific functional  
574 brain architecture.

575  
576 Our results extend this view by showing that these high-amplitude frames are closely  
577 linked to the topological organization of brain dynamics. Peak-dense pure nodes  
578 exerted a strong influence on the quality of Mapper modularity, and their selective  
579 perturbation substantially disrupted task segregation (as shown in Fig. 3). Their  
580 elevated within-task centrality further suggests that they act as organizing points around  
581 which task-specific brain states cluster.

582  
583 In prior work using node-based Mapper representations derived from resting-state data,  
584 hubs showed broad participation across canonical networks and were interpreted as  
585 integrative states shared across ongoing brain dynamics (Saggar, Shine, et al., 2022).  
586 The hubs identified here differ in both context and function: during task conditions, high-  
587 amplitude interaction patterns form task-specific anchors around which activity  
588 organizes. Rather than broadly integrating activity, these hubs stabilize the system  
589 within particular coordination regimes. Together, this suggests that dynamic brain  
590 organization involves complementary hub types, integrative hubs prominent in  
591 unconstrained dynamics and context-specific hubs that anchor coordinated states  
592 during goal-directed behavior.

### 593 594 **3.3 Phase structure revealed by edge representations**

595  
596 We next examined why edge and node representations produce different topological  
597 organization. We observed that edge-based distances follow an approximate quadratic  
598 transformation of node-based distances (Fig. 6), consistent with prior work suggesting  
599 that edge representations reorganize, rather than add to, the information contained in  
600 activation patterns (Novelli & Razi, 2022). In particular, we reproduce the observation  
601 that the edge time series tends to cluster anticorrelated time frames together (Betzel,  
602 Faskowitz, et al., 2023). Under this transformation, activation patterns that are  
603 oppositely signed but share the same interaction structure become nearby in edge  
604 space. Consequently, Mapper clusters time frames according to coordination structure  
605 rather than activation polarity.

606

607 This property provides a geometric explanation for the enhanced task segregation  
608 observed in edge Mapper graphs. Task-related brain states often involve coordinated  
609 but phase-opposed activity across networks, a phenomenon reported in studies of  
610 bimanual movement and inter-hemispheric communication (O'Reilly & Elsabbagh, 2021;  
611 Wu et al., 2010). By treating antiphase patterns as related states, the edge  
612 representation groups fluctuations that belong to the same functional regime while  
613 ignoring sign reversals that may arise from oscillatory or context-dependent activity. In  
614 this sense, the representation emphasizes phase-consistent coordination patterns,  
615 leading to more coherent topological structure.

616  
617 Together, these findings suggest that edge time series reveal a phase-based  
618 organization of brain dynamics that is obscured in activation space, providing a  
619 geometric account of their stronger task-dependent topology.

### 620 **3.4 Behavioral relevance of the quality of edge Mapper modularity**

621  
622 Topological properties of brain dynamics have long been linked to behavior, though  
623 most studies rely on linear summaries such as functional connectivity. Here, we found  
624 that the quality of modularity of the edge time series Mapper graph relates to individual  
625 differences in both temperament and symptom scales. This property may reflect the  
626 degree to which brain dynamics organize into well-separated coordination regimes over  
627 time.  
628

629  
630 Conscientiousness, a NEO-FFI domain associated with organization, planning, and  
631 goal-directed behavior (Costa & McCrae, 1992), showed a positive association with the  
632 quality of modularity. Prior work links conscientiousness to structural and functional  
633 properties of control-related networks, including lateral prefrontal cortex morphology  
634 (DeYoung et al., 2010) and goal-priority systems involving salience and ventral attention  
635 networks (Rueter et al., 2018; Sassenberg et al., 2023). In this context, higher quality of  
636 modularity may reflect more stable and selective engagement of coordinated neural  
637 states, consistent with efficient regulation of attention and behavior.

638  
639 Conversely, internalizing and externalizing symptoms were associated with reduced  
640 quality of modularity. These symptom dimensions have been linked to altered coupling  
641 among default mode, frontoparietal, and limbic systems (Kaiser et al., 2015; Thijssen et  
642 al., 2021) and to reduced segregation of large-scale networks. Lower quality of  
643 modularity in the present framework may therefore indicate less differentiated  
644 coordination patterns, consistent with diffuse or interfering neural dynamics.

645  
646 Together, these relationships suggest a continuum in which well-segregated  
647 coordination patterns correspond to organized, goal-directed behavior, whereas  
648 reduced segregation corresponds to behavioral dysregulation and inward distress.  
649 Rather than serving as a diagnostic marker, the quality of Mapper modularity may index  
650 the degree of compartmentalization in ongoing brain dynamics, a property that plausibly  
651 supports both behavioral regulation and emotional stability.  
652

### 653 **3.5 Limitations and Future Works**

654

655 This study has several limitations. First, our findings were derived from the Human  
656 Connectome Project dataset, which consists primarily of healthy adults. Although the  
657 large sample size allows detection of small brain–behavior associations, replication in  
658 independent cohorts and clinical populations will be important to establish  
659 generalizability and clarify relevance to psychiatric conditions.

660

661 Second, the interpretation of Mapper graphs remains challenging because they  
662 summarize very high-dimensional dynamics through a non-linear construction. While we  
663 provided geometric and dynamical explanations for the observed structures, the  
664 relationship between Mapper topology and underlying neural mechanisms is indirect.  
665 Future work could address this by applying the framework to simulated data with known  
666 ground-truth dynamics, such as biophysical network models (Deco et al., 2014;  
667 Demirtaş et al., 2019; Zhang et al., 2022) or network control theory models (Ceballos et  
668 al., 03 2025; Gu et al., 2015), enabling systematic evaluation of what properties of  
669 neural systems are recoverable from topological summaries.

670

671 Third, our comparison of interaction orders was limited to node-, pairwise-, and triplet-  
672 level representations. Higher-order interactions rapidly incur combinatorial growth when  
673 defined via time-wise products, making exhaustive evaluation computationally  
674 prohibitive. Thus, our results do not imply that higher-order interactions are absent, but  
675 rather that pairwise interactions provide a practical level at which stable structure  
676 becomes detectable with current data and representations.

677

678 Fourth, edge time series were defined using correlation-based co-fluctuations.  
679 Alternative definitions of interactions (e.g., nonlinear coupling or model-based  
680 interactions) may capture complementary aspects of brain dynamics and could alter the  
681 recovered topology. Exploring such formulations will be important for determining which  
682 features of neural coordination are most robustly reflected in topological  
683 representations.

684

685 Finally, our analyses were based on BOLD fMRI signals, which reflect neural activity  
686 indirectly through hemodynamic responses. The temporal smoothing imposed by the  
687 hemodynamic response function may influence the prominence of high-amplitude  
688 events and the apparent phase relationships captured by edge time series.  
689 Consequently, the topology described here should be interpreted as characterizing  
690 large-scale hemodynamic coordination rather than instantaneous neural interactions.  
691 Future work applying the same framework to electrophysiological recordings (e.g., MEG  
692 or EEG) will be important to determine which aspects of the observed topology  
693 generalize across temporal scales and which are specific to BOLD dynamics.

694

### 695 **3.6 Conclusions**

696

697 In summary, we demonstrate that the interaction order used to represent brain activity  
698 shapes the organization recovered from neural dynamics. Pairwise co-fluctuations

699 reveal a coherent topological structure that is driven by high-amplitude events,  
700 explained by geometric properties of the representation, and associated with stable  
701 behavioral variation. These findings position topological analysis as a useful framework  
702 for studying large-scale brain coordination and suggest that behaviorally relevant  
703 information may be encoded in the global organization of dynamic interactions rather  
704 than in regional activation alone.

705  
706

## 707 **4. Methods**

708

### 709 **4.1 The Human Connectome Project Dataset and Preprocessing**

710

711 We used the 3T resting state and task-based fMRI data, as well as demographics and  
712 behavioral data (sex, age, family, NEOFFI scores, and symptom scale scores) from the  
713 de-identified publicly available data from the Human Connectome Project Young Adult  
714 (HCP-YA) dataset (Barch et al., 2013; Smith et al., 2013; D. C. Van Essen et al., 2012;  
715 David C. Van Essen et al., 2013). As per the HCP protocol guidelines, all participants  
716 gave written informed consent for data collection. The HCP scanning protocol was  
717 approved by the local Institutional Review Board at Washington University in St. Louis.  
718 unrelated subjects who completed the resting state scan and 7 tasks (emotion,  
719 gambling, language, motor, relational, social, working memory) were selected to form  
720 the exploration cohort of 394 subjects. Another 319 unrelated subjects were selected to  
721 form the replication cohort. For behavioral correlation, we repeat the computations in  
722 the total cohort of 968 subjects, that comprises both the exploration cohort, the  
723 replication cohort, as well as siblings of subjects in both cohorts. For a fairer comparison  
724 between resting state activity comparable to task activity, we only include the first 5  
725 minutes of the resting state scan.

726

727 For preprocessing, we follow the pipeline described in (Saggar, Shine, et al., 2022),  
728 which we briefly describe below. Minimally processed data were first gathered from the  
729 HCP database. This minimal processing includes spatial normalization, motion  
730 correction, and intensity normalization (Glasser et al., 2013). We additionally processed  
731 these data using fMRIPrep 1.5.9 (Esteban et al., 2019).

732

733 fMRIPrep anatomical preprocessing included intensity non-uniformity correction with  
734 N4BiasFieldCorrection (ANTs 2.2.0), skull-stripping using antsBrainExtraction.sh  
735 (OASIS30ANTs template), and tissue segmentation into CSF, white matter, and gray  
736 matter using FSL FAST. Nonlinear registration to MNI152 standard spaces was  
737 performed with antsRegistration (ANTs 2.2.0).

738

739 fMRIPrep functional preprocessing included generating a skull-stripped BOLD reference  
740 volume, which was co-registered to the T1w reference using boundary-based  
741 registration (FSL FLIRT, 9 degrees of freedom). Head-motion parameters were  
742 estimated using MCFLIRT, and BOLD time-series were resampled to native space with  
743 motion correction applied. Confound time-series were calculated including framewise

744 displacement (FD), DVARS, and global signals from CSF, white matter, and whole-brain  
745 masks.

746  
747 Temporal masks flagged motion-contaminated frames using  $FD > 0.5$  mm, including  
748 one preceding and two following frames. Data were then (i) demeaned and detrended,  
749 (ii) regressed against whole-brain, CSF, and white matter signals plus Volterra-  
750 expanded motion regressors (excluding censored frames from beta estimation), (iii)  
751 interpolated across censored frames using linear estimation, (iv) band-pass filtered  
752 (0.009-0.08 Hz), and (v) censored frames removed for final analysis.

## 753 754 **4.2 Edge Time Series**

755  
756 The node, edge and triangle time series are formed as follows. We use the Schaefer  
757 100 x 7 parcellation scheme (Schaefer et al., 2018). Computationally, the *raw* node time  
758 series is a matrix where each entry is the average BOLD signal of the voxels in a parcel  
759 (column) at a time frame (row). We obtain the node time series by z-scoring the raw  
760 node time series across time parcel by parcel. The edge time series is a matrix where  
761 each column corresponds to a pair of parcels, and it is the timewise product of the  
762 corresponding columns in the node time series. The triangle time series is similar, with  
763 each column corresponding to a triplet of parcels.

764  
765 To combine different fMRI scans of the same subject, we construct such time series for  
766 each scan, and then we concatenate them. In particular, z-scoring is done within each  
767 task.

768  
769 For each instantaneous cofluctuation, which is a row of the edge time series, the  
770 amplitude is defined as the  $\ell_2$  norm of the row. Mathematically, suppose the parcellation  
771 scheme contains  $m$  parcels. Denote by  $C_{i,j}(t)$  the cofluctuation of parcel  $i$  and  $j$  at time  $t$ .  
772 The instantaneous cofluctuation vector is then

773  $C(t) = [C_{1,2}(t), C_{1,3}(t), \dots, C_{i,j}(t), \dots, C_{m-1,m}(t)]$ . The amplitude of  $C(t)$  is

774  $\|C(t)\|_{\ell_2} = \sqrt{C_{1,2}(t)^2 + C_{1,3}(t)^2 + \dots + C_{i,j}(t)^2 + \dots + C_{m-1,m}(t)^2}$ .

## 775 776 **4.3 Node-Based Distance and Edge-Based Distance**

777  
778 The node-based distance is simply correlation distance. The edge-based distance is  
779 correlation distance for the cofluctuation vectors.

## 780 781 **4.4 Mapper Graph**

782  
783 The Mapper graph is a graph where nodes correspond to clusters of data points, and  
784 edges connect nodes whose corresponding clusters overlap (Singh et al., 2007).  
785 Construction of the Mapper graph involves two main steps.

786  
787 First, filtering: data points are projected into a low-dimensional space using a nonlinear  
788 dimensionality reduction method. In this study, we use Isomap, where neighborhood  
789 relationships are defined using a k-nearest neighbor (kNN) graph with  $k = \sqrt{N}$ , where  $N$

790 is the number of data points. This choice balances preservation of local and global  
791 structure in the data.

792  
793 Second, local clustering: the low-dimensional space is partitioned into overlapping bins,  
794 and data points within each bin are clustered using a correlation-based distance metric  
795 computed from the original dimensional space. The resulting clusters form the nodes of  
796 the Mapper graph, and edges are placed between nodes that share data points due to  
797 overlap in the binning step.

798  
799 To reduce sensitivity to manual parameter selection, Mapper parameters were chosen  
800 using an adaptive, data-driven strategy (Quah et al., 2025). Specifically, the number of  
801 bins (resolution) was determined using kernel density estimation (KDE) to reflect the  
802 underlying data density, enabling finer partitioning in regions of higher concentration.  
803 The overlap between bins was selected through an iterative procedure to ensure that  
804 the largest connected component of the Mapper graph exceeded a predefined threshold  
805 ( $\geq 80\%$  of nodes), promoting a coherent and interpretable topological representation.

806  
807 A Mapper graph can be annotated using both categorical and quantitative metadata.  
808 For quantitative variables (e.g., amplitude), nodes can be colored by the average value  
809 within each cluster. For categorical variables (e.g., task identity), nodes can be  
810 represented using pie charts indicating the proportion of data points from each category.

## 811 812 **4.5 Centrality and Modularity**

813  
814 Given a graph (an object made up of nodes and edges), the closeness centrality of a  
815 node is the reciprocal of the sum of the distances from the node to all other nodes in the  
816 graph (Sabidussi, 1966). The within-task centrality of a node weighs a node by the  
817 proportion of time frame in the mode task of the node.

818  
819 For a graph with a partition of its nodes (grouping of the nodes such that each node  
820 belongs to exactly one group), the quality of modularity of the graph with respect to the  
821 partition, roughly speaking, measures the discrepancy between the proportions of in-  
822 group edges and inter-group edges (Newman, 2006; Rubinov & Sporns, 2010).

823 Precisely, consider a graph with  $n_V$  nodes. Denote by  $D_i$  the number of nodes  
824 connected to node  $i$ . For every pair of nodes  $i$  and  $j$ , let

$$825 \quad Q_{ij} = \mathbf{1}_{ij} - \frac{D_i D_j}{n_V},$$

826 where  $\mathbf{1}_{ij}$  is 1 if nodes  $i$  and  $j$  are connected, and it is 0 otherwise.

827 The quality of modularity  $Q$  of the graph with respect to a partition is the sum of the  $Q_{ij}$ 's  
828 for all in-group node pairs  $i, j$  (i.e.  $i$  and  $j$  belong to the same group).

829  
830 The shuffled quality of modularity of a graph with respect to a collection of nodes is the  
831 bootstrap mean of the quality of modularity with the node label (in our case, task labels)  
832 randomly shuffled.

## 833 834 **4.6 Thresholds of Peak Frames and Peak-Dense Nodes**

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A frame is said to be a peak frame if its amplitude exceeds the 95th percentile. A Mapper node is said to be pure if the proportion of the mode task is at least 75%. A pure Mapper node is said to be a peak-dense pure node if its proportion of peak frames exceeds the 90th percentile. We justify the choice of these thresholds in Supplementary Materials Section S6.

#### 4.7 High-Amplitude Functional Connectivity

The peak-frame functional connectivity is the temporal mean of confluctuation at peak frames. The peak-dense-pure-node functional connectivity is the temporal mean of confluctuation of frames that belong to peak-dense pure nodes.

#### 4.8 FDR correction

FDR correction is applied when the number of tests is large. Generally, Bonferroni correction is done independently for each table in the Supplementary Section. For results regarding personality, behavioral symptoms and fluid intelligence in Section 2.5, within each table (Tables S10 – S12), Benjamini-Hochberg correction is applied for each simplex (node, edge, and triangle) and each covariate control (none, demographics, etc). Further, after the initial analysis in the exploration cohort with no covariate control, features significantly correlated with the quality of modularity after FDR correction are singled out, and only those are tested in the subsequent analyses with covariate control, and subsequently in the total cohort. When no association survive FDR correction in the exploration cohort without covariate control, as in the case of triangle time series, we carry subsequent analysis with features that are significant without correction.

## 865 **Data Availability**

866 The data used in this work were originally collected as part of the Human Connectome  
867 Project (HCP) (Barch et al., 2013; Smith et al., 2013). We gathered these data directly  
868 from the HCP website (<https://db.humanconnectome.org>).

## 870 **Code Availability**

871 Codes to generate Mapper graphs are available at the Mapper repo  
872 (<https://github.com/braindynamicslab/demapper>). Besides codes in our repo, edge time  
873 series can also be computed with codes in the edge time series repo  
874 (<https://github.com/rbetzel/edge-time-series>).

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884  
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